

# Mathematical Knowledge Management for Enterprise Decision Making

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## Abstract

The basis of decision-making in the enterprise consists of formally representing the system and its subsystems in a model which adequately captures those features which are necessary to reach consistent decisions. This work aims at integrating the elements of the enterprise (i.e., decisions, parameters, constraints, performance indicators) which are included in mathematical models to a semantic representation of the enterprise by the creation of an ontology which captures the meaning of the mathematical language. As a result, the integration of decisions in enterprise may be achieved. The purpose of this work is illustrated in a case study related to plant capacity in the supply chain and scheduling problems.

**Keywords:** Mathematical Modeling, Decision Support Systems, Enterprise Integration, Knowledge Management.

## 1. Introduction

Nowadays, the application of analytical systems for decision-making has gained increased attention in enterprises since they can provide insights to find solutions that help businesses to remain competitive in the current environment of globalization of markets and fierce competition. The basis of decision-making consists of formally representing the system to be analyzed, in this case the enterprise and its subsystems, in a model which adequately captures those features that are necessary to reach consistent decisions.

In general, models can be classified according to the way that information is represented. Therefore, a general classification distinguishes between qualitative and quantitative models. The former represent the physical and logic relationships among the elements of the system and describe the reality, such as conceptual or semantic models; whereas the latter allow proposing decisions based on the analysis of actual data regarding the system, such as mathematical or statistical models.

Models adopted in enterprises are usually of high complexity. In addition, there is not a single valid model or model representation for each problem and the adoption of a model depends on both the decision-maker preferences and the problem features. As a

result, several models originally described with very different notations may be considered for representing a given problem. In this sense, semantic technologies support a smooth integration of descriptive and optimization modeling in a single framework. Therefore, the adoption of such technologies within industrial practice may facilitate the integration of existing PSE tools with the IT environment of enterprises.

This work aims at translating the elements (i.e., decisions, parameters, constraints, performance indicators) of enterprise mathematical models to a semantic representation by creating an ontology. Once this representation is developed, each concept behind such elements is unified/standardized into the classes, properties, and axioms of an existing ontology related to the enterprise domain (Muñoz et al., 2012). An important advantage of carrying out this translation is that it facilitates the integration of different models and the easily incorporation of other system aspects (e.g., constraints, performance indicators) as well. Some previous work exists along this line for manufacturing modeling in the pharmaceutical domain (Venkatasubramanian et al., 2006).

Instead, this work is focused on models for the strategic and operational levels of the enterprise. Specifically, a semantic model is constructed for the supply chain (SC) strategic design-planning model presented in the work of Laínez et al. (2009) and for the scheduling model presented in Capón-García et al. (2011). It is demonstrated how this translation is carried out and how the mathematical models can be recreated and their integration achieved.

Based on two proposed principles, namely freedom of mathematical reality conception and the semantic actor assumption of reality, generality in models aim to be achieved. As for the principle of freedom, this work does not intend to rule how the mathematical models must be constructed, maintaining the Engineers freedom of conception. This work aims at linking the different variables found in mathematical models, with the real aspects (reality) that they are representing. As a result, a framework that supports robust mathematical model integration is developed.

## **2. Integration problem**

Recent trends in process industries are shifting the focus from controlling the process plant as a stand-alone entity toward managing it as an integral part of a larger system (Klatt, 2009). Obviously, such understanding of process management entails the integration of the different decision level functions. Therefore, a current important challenge lies on the coordination of the decision making and the optimization of different decision levels. In fact, the border lines between the decision-making levels of the enterprise structure are often diffuse, and there are strong overlaps between planning in production, distribution or supply chain management and strategic planning.

One first step toward such integration consists of sharing of information, which is nowadays being achieved with modern IT tools. However, a better understanding, structuring and even modeling of the whole process is necessary for an effective transformation of the information into knowledge. Semantic technologies seem to offer an appealing way to capture knowledge and integrate information, for supporting a smooth integration of information and mathematical modeling in a single framework (Klatt, 2009).

Therefore, this work represents an additional step for the development of an integration framework, which effectively solves the information sharing among models, so that better overall solutions are reached.

### **3. Mathematical models in the enterprise**

This work considers the mathematical models posed in the literature regarding the supply chain and the scheduling decision levels. Next, their main features are briefly detailed.

#### *3.1. Supply chain models*

The concept of SC refers to the network of interdependent entities (i.e., retailers, distributors, transporters, storage facilities and suppliers) that constitute the processing and distribution channels of a product from the sourcing of its raw materials to its delivery to the end consumer. The major driving forces behind a SC are customer satisfaction, economic incentives, efficiency and risk. Economic drivers show how profitable it is, such as total cost, profit or net present value (NPV).

SC performance is an outcome of the decisions made in order to synchronize the materials, information and cash flows along the SC partners. The decisions encompassed in a SC model depend on its scope, among them facility location, capacity changes, flows magnitude, resource allocation or inventories. The management alternatives are characterized in the model restrictions such as mass balances; SC capacity constraints; technological constraints; budgeting limitations; suppliers' capacity; and market demand and competition among others. Several mathematical representations of the SC structure have been proposed in the literature with different levels of detail (Shapiro, 2004; Varma, 2007).

#### *3.2. Scheduling models*

The building block of batch process scheduling is the process recipe, which contains the whole information required to produce the product, as well as the set of processing tasks, i.e. the process flow. The recipe is usually obtained in a design stage prior to the scheduling stage, and the process conditions to perform the product are optimized and fixed for all production batches of a given product. In addition, such information must be complemented with the production facility data regarding equipment and resources, such as manpower, inventory or general services availability; production planning information regarding sales, time horizon, order due dates or prices; and actual plant state. As a result, the scheduling function determines the amount of each product to produce, the allocation of equipment and resources to tasks, as well as the sequencing and timing of such tasks, in order to fulfill certain objectives.

Different mathematical models have been proposed in the literature in order to adequately formulate scheduling problems. However, each modeling option is only able to cope with a subset of features. The choice of the mathematical model has an important impact on computational performance. Hence, the model capabilities and limitations must be carefully considered for each scheduling problem. Recent reviews describe the most used mathematical formulations and compare their features and performance (Shaik, 2006; Pan, 2009). In general, scheduling models may be classified according to time representation, material balance representation and event representation. Insomuch the previous formulations have been continuously extended to

consider additional process scheduling features, some uncommon features which are highly case dependent should still be tackled.

## 4. Proposed knowledge management approach

### 4.1. Mathematical ontology model

In this section, the knowledge management tool for the translation, interpretation and management of mathematical models is presented. The basis consists of a semantic model which relies on the Mathematical Markup Language (MathML), an XML application for describing mathematical notation. The semantic model captures both the structure and content of the mathematical expressions, and it is capable of relating the mathematical elements to the semantic representation of the enterprise.

The "Ontological Math Representation" approach focuses in the semantic representation of the mathematical content of functions and equations. The mathematical expressions are represented as mathematical sets that are related among them by ontological properties. Thus, it handles the expression of functions, the quantification and the operators with qualifiers. The resulting chains translate the mathematical expressions into the process explicit reality of the enterprise which is easier to understand by humans and machines.

In addition, the proposed semantic model includes the treatment of bounding variables. Such variables are central representational primitives in mathematical languages improving the knowledge management in this domain. Besides, the various bounding variables are related to the enterprise objects by the connection with the process enterprise ontology (Figure 1).

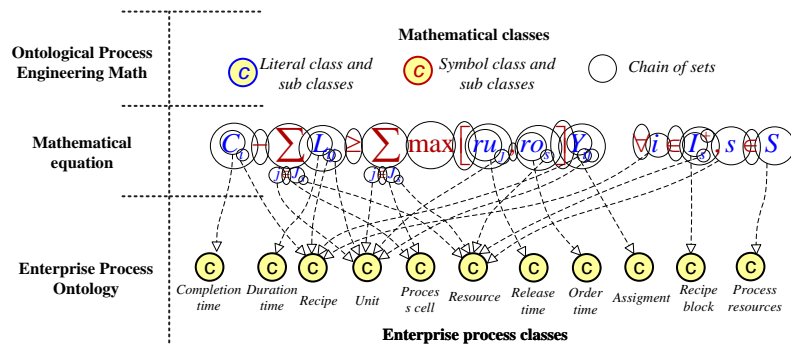


Figure 1. Representation and interpretation of a mathematical equation using the ontological process and the enterprise process ontology.

Even more, the approach allows the integration of those mathematical models used as decision support tools at different hierarchical levels in the enterprise. This integration is possible because every variable is related to an ontological reality (OR). This OR is a semantic standardization of terms for a consensual understanding.

### 4.2. Enterprise process ontology model

In a previous work (Muñoz, 2011), the Enterprise process ontology model was presented. Such model contains an integrated representation of the whole enterprise

structure, ranging from the supply chain management to the scheduling function and comprising activities related to the operational, tactical and strategic functions. The model is based on the understanding and management of operational concepts (physical models, procedures, functions and processes) provided by process standards, and complemented with other handbooks and reviews.

The resulting integration process between the mathematical models used in two hierarchical levels is shown in Figure 2. Specifically, the supply chain and scheduling mathematical models are merged when the respective equations are interpreted and related to the classes of the enterprise process ontology model. As a result, an integrated mathematical model is obtained and can be solved by the software capable of reading xml language. Even more, in the classical hierarchical approach, this framework allows tracking the use of the elements of the different models and sharing the information obtained from the optimization processes, which may be performed separately.

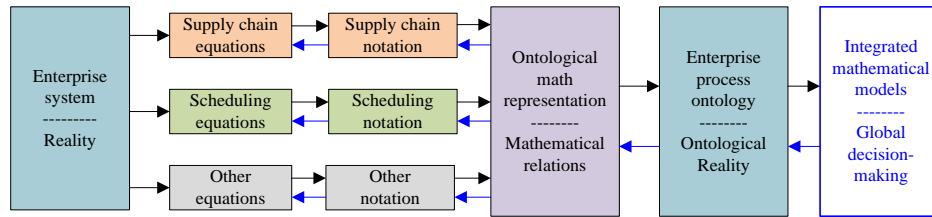


Figure 2. Scheme of the information flow between the enterprise system, its representation in mathematical equations and their interpretation to semantic model.

## 5. Case study and results

The semantic models of the mathematical formulations presented by Laínez et al. (2009) related to the supply chain strategic design-planning model and by Capón-García et al. (2011) related to the scheduling level are built using the "Ontological math representation". Thus, in this process the bounding variables are identified and used for model integration and variable tracking. Specifically, two main variables relate both formulations: (i) the equipment capacity of the plant at the operational level ( $BS_u^{max}$ ) is determined at the strategic level as the capacity installed in the plant ( $JF_{jf}$ ) (Eqs. (1) and (3)); (ii) the demand to be fulfilled by the plant ( $D_i$ ) is determined by the strategic model ( $SL_{sff't}$ ) (Eqs. (2) and (4)). In these equations, it is clearly seen that each formulation uses their own nomenclature and notation; however, they refer to the same terms. Hence, the proposed semantic model allows for transparency between models, enhancing communication and information exchange.

### Equations of the SC formulation

$f, f'$ : facilities,  $j$ : equipment,  $t$ : time period,  $s$ : states,  $e_f$  supplier sites,  $m_f$  market,  $JF_{jf}$  equipment  $j$  that can be installed in facility  $f$ ,  $SL_{sff't}$  sales of product  $s$  at time period  $t$  produced in facility  $f$  to market  $f'$ ,  $P_{ijf't}$  production of activity  $i$  in equipment  $j$  at period  $t$  produced in facility  $f$  to market  $f'$ ,  $O_{is}$  task  $i$  that produces state  $s$ ,  $JJ_{ji}$  equipment  $j$  that performs task  $i$ ,  $Tr_i$  distribution task,  $\alpha_{sij}$  capacity consumption of technology  $j$  of state  $s$  in task  $i$

$$FJ_{jft} = FJE0_{jf}|_{t-1} + FJ_{jft-1}|_{t-1} + FJE|_{jft} \quad \forall f, j, t | \neg e_f \cup \neg m_f \cup JF_{jf} \quad (\text{Eqn.1})$$

$$SL_{sff't} = \sum_{(i,j) \in O_{is} \cup Tr_i \cup JJ_{ji} \cup JF_{jf}} \alpha_{sij} \cdot P_{ijf't} \quad \forall s, f, f', t | \neg e_f \cup \neg m_f \cup m_f \quad (\text{Eqn. 2})$$

### Equations of the scheduling formulation

$i$ : materials,  $n$ : time events,  $p$ : products,  $s$ : stages,  $u$ : units,  $PU_{psu}$ : relationship between product, stage and unit,  $FP_i$  material which is final product,  $D_i$  demand of product  $i$ ,  $BS_u^{max}$  maximum capacity of unit  $u$ ,  $w_{psn}$ : binary variable stating that product  $p$  is produced at stage  $s$  in time element  $n$ ,  $SST_{in}$ : storage state of material  $i$  at time event  $n$ ,  $Bf_{psn}^f$ : batch size for product  $p$  and stage  $s$  at time event  $n$

$$Bf_{psn}^f \leq BS_u^{max} w_{psn} \quad \forall p, s, u, n | PU_{psu} \quad (\text{Eqn. 3})$$

$$SST_{in} \leq D_i \quad \forall i, n | FP_i \cup n = N \quad (\text{Eqn. 4})$$

## 6. Conclusions

Although progress dealing with mathematical modeling knowledge management such as data sharing among various solvers and standardized markup languages for model representation has been done, there is still the need for developing friendly integrated frameworks. The integration of mathematical models from different hierarchical levels (SC models and scheduling models) has been successfully done. This provides an additional degree of freedom for the enterprise modeling. So far the enterprise ontology relied on rigid analytical models which cannot be easily modified according to the new reality circumstances (constraints, variables). The new feature developed in this work opens the ability of modifying the analytical models from the ontological framework, providing a higher flexibility. In addition, the task of relating temporality was performed in order to match the two mathematical models. In this work, the ontological math representation, working together with the enterprise process ontology, stands for a better use of the mathematical models used as decision tools at the enterprises.

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