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Integration of Mathematical Programming and Game Theory Optimization in Multi-objective competitive scenarios

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Abstract

This work develops a multi-objective MILP (Mixed Integer Linear Programming) model, devised to optimize the planning of supply chains using Game Theory optimization for decision making in cooperative and/or competitive scenarios. The model developed is tested in a industrial case study, based on the operation of two different supply chains; three different optimization criteria are consider (total cost, tardiness and expenses of the buyers for the competitive problem), the multi objective problem has been solved using the Pareto frontier solutions, and both cooperative and non cooperative scenarios between supply chain´s is considered. Multiple optimization tools/techniques have been used in this work (Game Theory, MILP based approach and Pareto frontiers).

Keywords: Supply Chain Management, Planning, Multi-objective Optimization, Game Theory.

1. 1. Introduction

The problem of decision making in the process industry becomes more complex due to the need to consider multiple criteria and several considerations of the different departments of the network (production sites, storage centers and final consumers). This complexity is additionally complicated by the need to consider more sources of uncertainty in the models used to predict/control the events that should be considered in this decision making. The problem of decision making associated to supply chain (SC) operational management (procurement of raw materials in different markets, allocation of products to different plants and distributing them to different customers), which is attracting the attention of the scientific community in the last years, is in this sense, on the top level of complexity.

Multi objective optimization becomes an important tool to improve the decision making in the problems that has some tradeoffs between objectives, this tool can give many optimal solutions, which in turn provide greater degree of accuracy to the decision making.[1] Zamarripa et al., 2011 introduces the use of game theory as a decision technique that determines the optimal SC production, inventory and distribution levels in a competitive planning scenario, and models the competition behavior of several SC's as an uncertainty source, setting that the markets are embedded in a competitive market and models this problem taking into account the decisions of the others SC's, since these decisions impact to the profit of their own SC.

In order to deal with the complexity associated by the competition of the markets and keep looking to improve the decision making under multiple objectives, this work proposes to develop a multi-objective MILP (Mixed Integer Linear Programming), to

optimize the planning of SC in a competitive/cooperative environment, where decisions are inventory, distribution and productions levels, for several Supply Chains. Different objectives are optimized using the Pareto solutions, these solutions are obtained based on the normalized normal constraint method (Messac et al., 2003) to represent the payoff matrix that allows us to choose the Nash equilibrium (John Nash 1950), which is the best solution for several scenarios of the problem.
The cases of the competition SC are analyzed using a MILP model solving multiple objectives and tools from non-cooperative Game Theory; to highlight the results an eventual cooperative work is analyzed.

2. 2. Problem statement.

2.1 Supply Chain planning (cooperative and competitive scenarios)

The planning problem is typically to determine the optimal production, storage and distribution variables in a SC network of production sites, distribution centers, costumers, etc. The mathematical formulation for this problems typically leads in a mixed integer linear programming (MILP), and the solution determines the optimal values of the variables mentioned above.

The model originally proposed by Zamarripa et al. (2011) has been adopted as a basis for the formulation presented in this paper, which will be complemented with additional constrains and will seek to minimize different Objective functions according to the considered scenario. This formulation assumes the existence of several supply chains that may work in cooperative or competitive scenarios. In both cases, the mathematical constrains associated to the model will be the same.

In the cooperative scenario the problem is formulated considering that the different SC acts as one and minimizes the total cost of the overall SC and tardiness of deliver the products to the consumers (multiple objectives).

Minimize the total cost:

$$Min\ z1 = \sum_{i \in I\_G(i,g)} \sum_n \sum_h a_{in} Q_{inh} (1+e_b)^h + \sum_{sc} \frac{1}{sc} \sum_{i \in I\_G(i,g)} \sum_n \sum_h c_{in} W_{scinh} (1+e_b)^h + \sum_{i \in I\_G(i,g)} \sum_n \sum_h d_{in} E_{scinh} (1+e_b)^h + \sum_{i \in I\_G(i,g)} \sum_n \sum_h \sum_j k_{inj} T_{scinhj} (1+e_b)^h \quad (1)$$

Minimize tardiness:

$$Min\ z2 = \sum_{i \in I\_G(i,g)} \sum_n \sum_h \sum_j [\frac{u_{inj}}{s_{inhj}}] T_{inhj} \quad (2)$$

In the GT the players (suppliers) can consider two types of games: zero-sum and nonzero sum. This article uses the nonzero-sum game, since the SC of interest will not try to maintain the overall benefit of the system; the strategy is implemented through a payoff matrix, which is made up by the different potential strategies and shows the behavior for each action of the SC against the actions of its competitors.

To play this game (competition behavior), each player should deal with the demand that customers really offer to him (from the total demand), and this can be managed basically through their service policy: prices and delivery times. So, additionally to the cost of the supply chains, it is necessary to introduce as an objective the reduction of the buyers' expenses (cost for the distribution centers). This has been done through the price rates ($Prate_g$), thus to play with the prices associated at the source and the destiny of the products, Eq. (3).

$$Min\ CST(g) = \sum_{i \in I\_G(i,g)} \sum_n \sum_h \sum_j Ps_{inj} T_{inhj}\ Prate_g + z1 \quad (3)$$

Multi objective optimization (MOO)

The MOO is better known as the multi criteria optimization, this kind of problems considers multiple criteria to optimize the same decisions variables, and the same constrains of the problem. In a convex problem the solution of this optimization suppose a better solution. This optimization tool is widely used in chemical engineering problems as it gives us flexibility to evaluate our model for situations that bring benefits to both objectives (maximize benefit and minimize environmental issues).

The general representation of the multi-objective problem is:

$$minimize\ f_1(x)$$
$$minimize\ f_1(x)$$
$$s.t.$$
$$x^L \leq x \geq x^U$$
$$h(x) = 0$$
$$g(x) \leq 0$$

This work use the Pareto solutions that are obtained by the normal constraint method, this solutions take place into the payoff matrix, this matrix shows the different solutions for each scenario of the competitors and let us to choose the best solution for the problem (the Nash equilibrium point).

3. Case study

These concepts have been applied to a case study previously adapted by Zamarripa 2011 and based from (Wang and Liang, 2004, 2005 and Liang 2008). There is a factory that tries to maintain the work force level over the planning horizon, and supply as much product as possible (a fixed demand), some inventory levels are considered. The process consists in two products (P1 and P2), with a market demand for 3 months (time horizon) this demand become from 4 distribution centers (Distr1 to Distr4). Some extra data (initial storage, maximum and minimum production, distribution capacities, etc.), distribution network (figure 2) can be found at http://cepima.upc.edu/papers/MOCompetitive_SCs.pdf (tables 3-6).

4. Case study Results.

To compare the different supply chains considered in the problem the standalone solutions for each SC are shown in the table 1, also in the same table can be found the comparison with the original multi objective solution (Liang 2008).

Table 1: Comparative results between SC (original case and standalone cases)

	SC1 Liang 2008	SC1 Original data	SC1 standalone	SC2 standalone
Obj. Funct.	min z1+z2	min z1+z2	min z1+z2	min z1+z2
z1($)	788 224	719990	838652	840904
z2(hours)	2115	1887	1700	1747
Benefit ($)		3784060	3665347	3663095

CST ($)		5223939	5342652	5344904

The optimal solution for SC1 (standalone) is driven by the geographical conditions (nearest delivery), although different solutions are obtained according to the specific objectives considered. Differences between SC1 and SC2 standalone solutions are associated to the different distances from the production sites of SC2 to the markets. Detailed results can be found at http://cepima.upc.edu/ papers/MOCompetitive_SCs.pdf (Figures 2 and 3, and Table 7).

The solutions obtained for the cooperative case (when SC1 and SC2 work together to achieve the same market demand) are shown in the figure 3 that consists in the Pareto solutions for the multi objective problem (tardiness vs total cost).

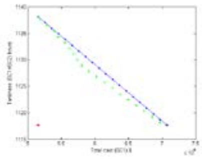
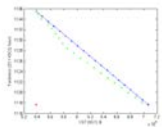
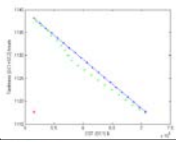
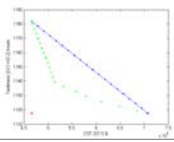
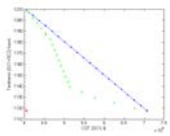
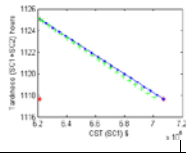
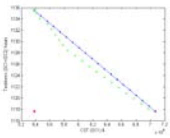
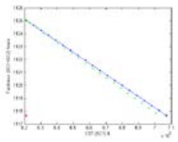
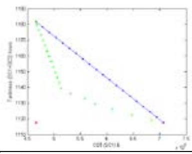
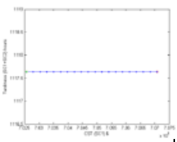
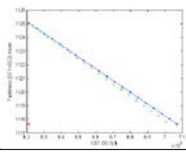
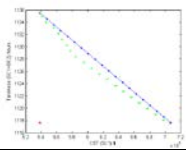
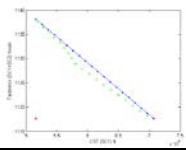
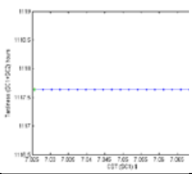
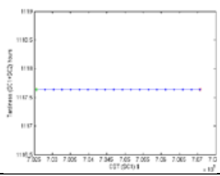
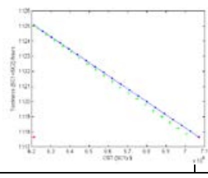
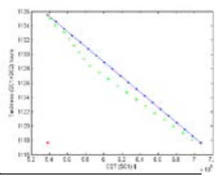


Figure 1. Pareto solutions for the cooperative case.

The figure 3 consists in the anchor points (that represents the best optimal solutions for each objective), the Utopia point ("*", the union of both objectives, this solution do not exist), the Pareto frontier (the frontier is constructed by drawing a line between the anchor points and divided into "mk" points) and the Pareto solutions "+" (the best solution is the one that is closer to the utopian point).

Table 2. Payoff matrix (discount percent)

SC1\SC2	0 %	0.1 %	0.2 %	0.3 %
0 %				
0.1 %				
0.2 %				
0.3 %				

For the competitive case, the model take into account the consumers preferences (these preferences have been modelled as just based on service and customers cost. A nominal selling price has been introduced to maintain the data integrity. The payoff matrix (Table 2) is built with the solutions obtained from the Pareto frontier for each scenario of the problem. The Nash equilibrium point of the payoff matrix represents the best solution of the non-cooperative problem (Table 3).

Table 3. Nash Equilibrium of the payoff matrix

	SC1	SC2
Discount	0.1	0
Obj. Funct.	min CST	
z1($)	621642	181203
Total cost	802845	
z2(hours)	1124	
Benefit ($)	2866066	831596
CST ($)	4109351	1194003

5. Conclusions

This work uses the GT as a tool for decision making that determines the optimal production, inventory and distribution in the SC planning problem. The cooperative and non cooperative multi objective problem has been modeled and solved using mathematical programming techniques (MILP models) and GT optimization strategies as the payoff matrix and the Nash equilibrium point.

The problem introduces the use of metric robustness, remarking the use of competitors as a source of uncertainty in typical SC planning problems. Also, the work considers the use of different methodologies to improve the decision making associated to the new challenges of the present and future industry problems (fewer inventories, more competition, more production capacity, etc.)

References

M. Leng, A. Zhu, 2009, Side payment contracts in two person nonzero sum supply chain games: review, discussion and applications. European Journal of Operational Research, 196, 600-618.

Y. Wang, 2006, Pricing production decisions in supply chains of complementary products with uncertain demand, Operations Research 54, 1110-1127.

T. Liang, 2008, Fuzzy multi-objective production/distribution planning decisions with multi product and multitime period in a supply chain, Computers and industrial Engineering 55, 678-694.

Wang, Liang, 2004, Application of fuzzy multi objective linear programming to aggregate production planning, Computers and industrial Engineering, 46, 17-41.

Wang, Liang, 2005, Applying possibilistic linear programming to aggregate production planning. International Journal of Production Economics, 98, 328-341.

Miguel Zamarripa, Adrian Aguirre, Carlos Mendez, Antonio Espuña, 2011, Improving supply chain management in a competitive environment, 21 European Symposium on Computer Aided Process Engineering- ESCAPE21, Elsevier, 1000-1005.

[1] Complementary material can be found at: http://cepima.upc.edu/papers/MOCompetitive_SCs.pdf

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